

3.2 Correlated Spike Trains

(Video 3.2) When correlations between spike trains are nonzero, STDP can create additional competition between groups of synaptic inputs. Let us first see how we can implement such correlations in spike times. There are various ways to create correlations between spike trains, while maintaining Poisson-like statistics within the spike trains themselves. For example, here we are going to create Poisson spike trains that have instantaneous and exponential correlations with each other. We discuss one method, for creating such correlations, which has been described by Brette and colleagues [1] (Method II).

Consider a Poisson spike train S_{source} with firing rate r_{source} . One way to create another spike train that has nonzero spike-time correlations with S_{source} is to simply copy spike times from S_{source} to the new spike train, which we call S_{new} . If the spikes are copied with probability p , then the firing rate of any new spike train will be $p \times r_{\text{source}}$. More generally,

$$r_{\text{new}} = \sum_{k=1}^M p_k r_k, \quad (1)$$

for M different source spike trains. In this example, we just stick to one source spike train, from which we create multiple new spike trains. If $p = 1$, S_{new} is simply identical to S_{source} . However according to equation 14, for any $p < 1$, thinning of a Poisson spike train will result in $r_{\text{new}} < r_{\text{source}}$. What if we want the spike trains to have the same or higher firing rates? In order to compensate for the loss of spikes, the firing rate can be increased to reach r_{target} by adding additional spikes. This is done by superimposing another Poisson spike train S_{noise} onto S_{new} , with firing rate $r_{\text{noise}} = r_{\text{target}} - p \times r_{\text{source}}$. If this process is repeated independently for a number of S_{new} with the same p , S_{source} , and independent S_{noise} for each spike train, one obtains a group of correlated spike trains that each fire with Poisson statistics and firing rate r_{target} . Now, what are the correlations between such spike trains?

The cross-covariance between any two spike trains S_i and S_j is

$$CCVF(s) = \langle S_i(t)S_j(t+s) \rangle - \langle S_i(t) \rangle \langle S_j(t) \rangle \quad (2)$$

The angular brackets denote an average over time. This function is 0 for two independent Poisson spike trains, and is $r\delta(s)$ for two identical Poisson spike trains with firing rate r (this is also called the autocovariance).

The cross-covariance function between two new spike trains generated from the same source spike train by the thinning method described above, is

$$CCVF(s) = p^2 r \delta(s) \quad (3)$$

with $r = r_{\text{source}} = r_{\text{target}}$. The area under the cross-covariance function, $p^2 r$, normalised by the firing rate r , is the correlation c . We see here that $c = p^2$. Therefore, if we choose $p = \sqrt{c}$, the correlations between the spike trains will be c . The method described above generates instantaneous correlations between spike trains. You can also make exponential cross-covariance between spike trains. In order to do so, you can shift the spikes randomly after copying them from S_{source} . Randomly shifting the spikes does not change the properties of single spike trains: they remain Poisson and their firing rate does not change. Choose the random shift from an exponential probability density $P(x)$ for every spike.

$$P(x) = \frac{1}{\tau_c} e^{-\frac{x}{\tau_c}} \quad (4)$$

In this case, the cross-covariance function is a convolution of the probability density function:

$$CCVF(s) = \int P(x)P(x+s)ds = \frac{1}{2\tau_c} e^{-\frac{|s|}{\tau_c}} \quad (5)$$

We will refer to this type of correlation as exponential correlation. The variable τ_c controls the width of the correlation. First let us make groups of spike trains that have instantaneous correlations, without random spike time shifting. Create a group of 10 instantaneously correlated spike trains with $c = 0.1$, and a group of 10 instantaneously correlated spike trains with $c = 0.1$ and Show the cross-correlogram (the cross-correlation histogram over different time lags) for each group; the correlation structure should be visible there. Within groups, you should see a large peak at zero lag, which should be absent when taking cross-correlations between groups.

Now apply exponential cross-correlations, by using $\tau_c = 20$ ms. How has the cross-correlogram changed? You should now see an exponential-like decay from both sides of the peak at lag zero.

In the next unit, we will apply these correlated spike trains to the neuron model with STDP.

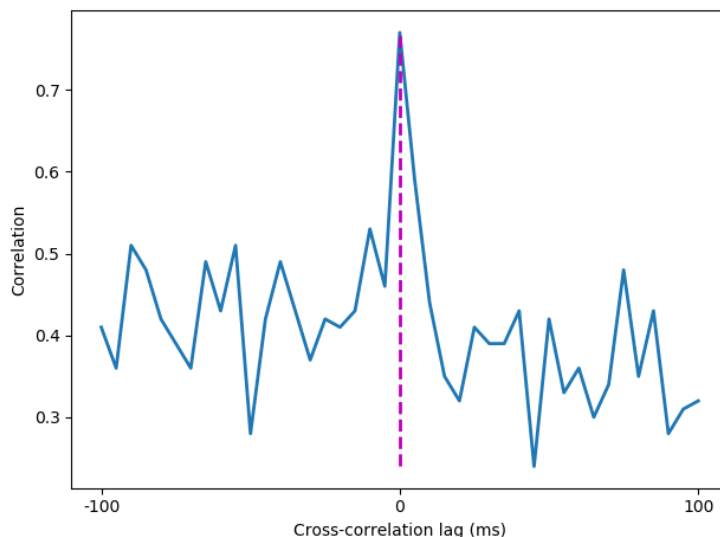


Figure 1: An example of a cross-correlogram between two 10 Hz Poisson spike trains of length 15 seconds. The spike trains are correlated with $c = 0.2$, and spike correlation jitter $\tau_c = 20$ ms. Spikes are binned in 5 ms bins. The vertical dotted line indicates the zero lag point. Although the spike correlation is jittered and not instantaneous, a peak can still be seen at zero lag.

References

1. R. Brette, "Generation of correlated spike trains.," *Neural computation*, vol. 21, pp. 188–215, jan 2008.