1.4 Poisson spikes

(Video 1.4) We now consider that the spikes the neuron receives resemble the irregular patters that are recorded in experiments. For example, a frequently used model for spike trains is the Poisson process. This means that spikes occur independently of each other. If the average firing rate remains stable over time, it is called a homogeneous Poisson process, which is what we will apply here. Since a property of a Poisson spike train is that the inter-spike intervals are distributed exponentially, a sequence of input spike times can easily be obtained by randomly sampling inter-spike intervals from an exponential probability density function.

Now expand the neuron model with N_e =10 excitatory and N_i =10 inhibitory inputs, with Poisson spike trains, each with a firing rate (intensity) of 10 Hz. In this model of multiple excitatory and inhibitory inputs, the equations now become:

$$\tau_\mathrm{mem}\frac{dV}{dt} = E_\mathrm{leak} - V + \sum_{m=1}^{N_\mathrm{e}} g_{\mathrm{e},m}(E_\mathrm{e} - V) + \sum_{n=1}^{N_\mathrm{i}} g_{\mathrm{i},n}(E_\mathrm{i} - V), \tag{1}$$

$$\frac{dg_{\mathrm{e},m}}{dt} = -\frac{g_{\mathrm{e},m}}{\tau_\mathrm{e}} + w_{\mathrm{e},m}\sum_{s=1}^{N_m}\delta(t - t_s), \tag{2}$$

$$\frac{dg_{\mathrm{i},n}}{dt} = -\frac{g_{\mathrm{i},n}}{\tau_\mathrm{i}} + w_{\mathrm{i},n}\sum_{p=1}^{P_n}\delta(t - t_p). \tag{3}$$

When the neuron receives excitation and inhibition, and excitation and inhibition are balanced, the output of the neuron should be irregular [1]. You can measure the irregularity of the output spikes by plotting the inter-spike intervals (ISIs). If the distribution of ISIs follows an exponential shape, and if the coefficient of variation (CV, the standard deviation/mean) of the ISIs is 1, the output is irregular. A perfect Poisson process has exponentially distributed ISIs, and a CV of the ISIs of 1. If the CV of the ISIs deviates from 1, this can be a sign of more regular firing or bursting.

Start with setting all $w_{\mathrm{e},m} = w_{\mathrm{i},n} = 0.5$. Is the spiking output of the neuron irregular? It will be necessary to adjust w_e or w_i to generate a more/less regular output from the neuron. Since the τ_i is larger than τ_e, you will need to increase w_e a little to compensate for inhibition and obtain excitation/inhibition balance in the input to the neuron. Show plots of the neuronal membrane V, spike times and show the distributions of ISIs and CVs of the ISIs. Insert the mean of the histograms in the plot titles. The number of ISIs in each trial is related to the output firing rate. The output firing rate, in turn, is mainly shaped here by the strength of the input weights and firing rates. Generally speaking, input spike timing also plays a role, but we will not address this yet, as here all input spike trains are Poisson with stationary firing rates. Make sure to run a sufficiently long simulation or a number of separate shorter runs ('trials'), so that you have enough ISI datapoints to clearly see the distribution of ISIs. For the CV of the ISIs, try to obtain at least 50 independent trials of 10 seconds each, so that you have 50 CV datapoints for the distribution, where each CV is based on at least 20 ISIs.

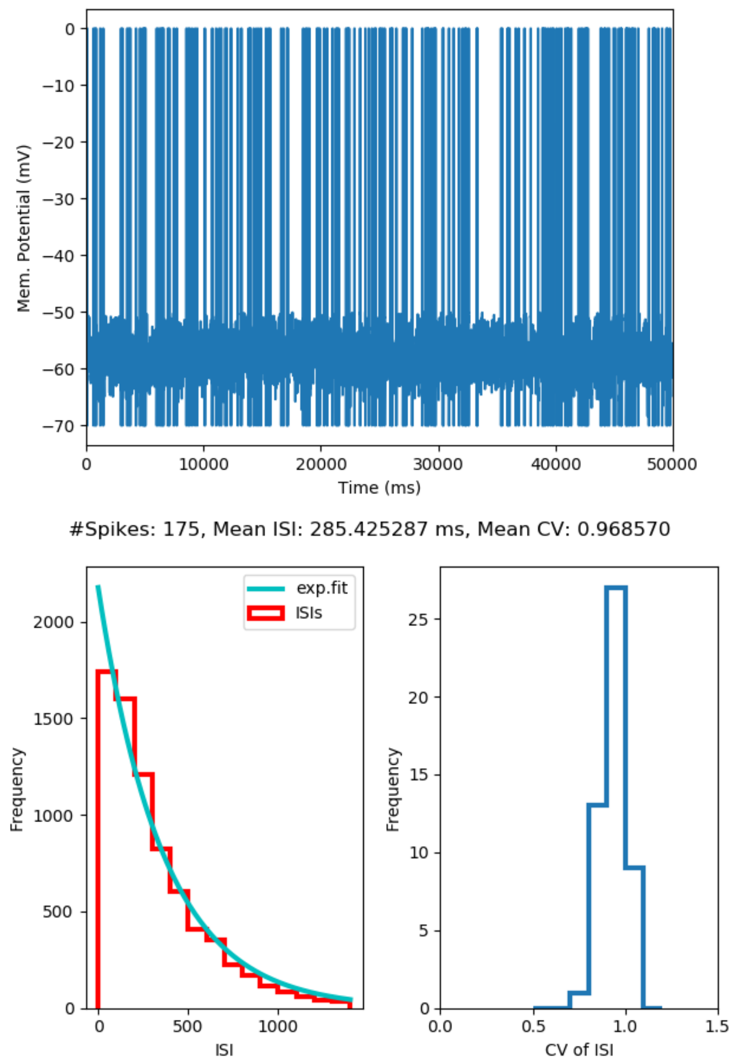

Figure 1: The LIF neuron with Poisson distributed synaptic inputs. Top: The LIF membrane potential shows irregular-looking fluctuations due to the Poisson input spikes. Bottom left: The distribution of inter-spike intervals (ISIs) of the LIF neuron after repeating 50 trials with Poisson inputs resembles a decaying exponential, indicating the irregularity of the output spikes. The exponential fit is found using `curve_fit` from Scipy. Bottom right: The CVs of the ISIs from separate trials, shown here in a histogram, are distributed close to 1.0, which would be expected for a Poisson process. One can obtain a CV close to 1.0 by balancing excitation with inhibition onto the neuron.

References

1. M. N. Shadlen and W. T. Newsome, "The variable discharge of cortical neurons: implications for connectivity, computation, and information coding.," *The Journal of neuroscience : the official journal of the Society for Neuroscience*, vol. 18, no. 10, pp. 3870–3896, 1998.